Invariant equivalents and their representation in the symbolic musical notation: a case in the musical culture of Paraguay

Victor M. Oxley
Profesor e investigador
National University of Pilar, Paraguay

Abstract. Understanding musical notation as a language and a system of musical communication brings a series of interesting ontological consequences. In this context and its consequent semantic theory it is possible to analyze how the musical discourse is articulated around its own and very singular referents. It is because of the open texture of language, the ambiguity of its terms, and from here, the ease of understanding the meaning of what is expressed under its rules of construction —in this case the musical grammar— can generate referential misses with unsuspected consequences from the point of view of musical significance. Taking the mathematical concept of "invariant" and corresponding to their structural equivalents in music, we can demonstrate how these can be modeled in different ways; these models will be very different in their formalism but equivalent as the object referenced (invariant). It cannot be categorically stated —as some Paraguayan music masters do— that there is a correct way of writing music (in this research the example is given within the cultural context of Paraguayan popular music but can be extended to any field) because in the conventional nature of writing, and understanding as a medium that refers to something.

Keywords. Semantic, Invariant, musical notation.

Resumen. Comprender la notación musical como un lenguaje y un sistema de comunicación musical trae una serie de interesantes consecuencias ontológicas. En este contexto y su consecuente teoría semántica, es posible analizar cómo el discurso musical se articula alrededor de sus propios y muy singulares referentes. Debido a la textura abierta del lenguaje, la ambigüedad de sus términos y, a partir de aquí, la facilidad de entender el significado de lo que se expresa bajo sus reglas de construcción —en este caso la gramática musical—, se pueden generar errores referenciales con consecuencias insospechadas, desde el punto de vista de la signifycación musical. Tomando el concepto matemático de “invariante” y respecto a sus equivalentes estructurales en la música, podemos demostrar cómo se pueden modelar de diferentes maneras; estos modelos serán muy diferentes en su formalismo, pero equivalentes como el objeto al que se hace referencia (invariante). No se puede afirmar categóricamente —como lo hacen algunos maestros de la música paraguayos- que existe una forma correcta de escribir música (en esta investigación, el ejemplo se da dentro del contexto cultural de la música popular paraguaya, pero puede extenderse a cualquier campo), dada la naturaleza convencional de la escritura y la comprensión como un medio que se refiere a algo.
Palabras clave. Semántica, invariante, notación musical.

Introduction
The composition “Jha Che Valle” of Agustín Barrios [San Juan Bautista de las Misiones-Paraguay, 1885 – San Salvador, 1944], along with the spaniard Francisco Tárrega, the greatest composer and interpreter of the classic guitar of the twentieth century, perfectly illustrates the phenomenon that is categorized under the label of “rhythmic illusions”; the predominant measure in the work is that of 6/8, but in the bars 36 (see example in the transcription of Jesus Benites paragraph below),

where the trio begins, it is verified that under the quadruple grouping on the original measure an illusion of change of metric is generated by the pulsation of the quadruples which converts to rhythm in the rhythmic illusion of 4/4 (see example: Figure # 2, in the transcription of Victor Oxley paragraph below).

---

3 BENITES R., Jesús: A. Barrios Mangoré, Zen On Music, Japan, 1974, p. 44.
Even in the music of the seventeenth to the nineteenth centuries the compass indications and the compass bars always faithfully reflect the actual metrical organization. Sometimes the composers used them in a somewhat informal manner, such as comfort, letting the performer interpret. For example, although the last movement of Schumann’s Piano Concerto in the minor is written at 3/4 the melody that enters at 80 is so strongly binary at the level. Although the previous ternary compass continues in the mind and in the motor response of the listener as well as the performer, making the music appear somewhat more tense-like hemiola-, the New compass is more like 3/2 (3 x 2/4) than the previous organization, 6/4 (2 x 3/4).

In jazz it is usual to encounter this type of phenomenon, by the very nature of its general Philosophy; it is common to listen to passages interpreted by a soloist without rhythmic support, generating this in which he hears a rhythmic identification that comes from the accentuation product of the figuration used and then when the other members of the combo adhere, there is the surprise that our rhythmic identification fades into context in another metric, now product of group synergy. These rhythmic illusions emerge when some small changes are introduced in a conventional rhythm, ex profeso to excite the spectator that something else is happening.

**Musical semantic**
Categorically saying that there is a 'right' way to write music x and is in 6/8 measurement, brings a series of interesting consequences to analyze about certain aspects of musical art.

---

Talking about writing music in a 'right' way implies semantically that there is a context in which you can demonstrate that there are 'wrong' ways of doing so.

You can go by drawing a line of synonyms or interchangeable terms, so you can also use the word 'successful' and get to the key term 'true' (with which the 'false' is associated). It is very common to exchange all these terms in the discourse.

In this way, the term 'correct' can easily be interchanged in the discourse with that of 'true', but this linguistic maneuver—which sneaks below—generates confusion and leads to conceptual misunderstandings. So much can this be that there is only a small step to the affirmation—for example—that 'everything that does not fit this true interpretation sins of falsity'.

The above statement is based on a confusion as to the ontology of space-time and the modeling of this in the symbolic musical notation [on the geometric model of music can be consulted Tymoczko (2011)].

Before advancing this analysis it is important to elucidate certain phenomena that make sense and reference of musical language.

The sign \( x \) designates the concept (or proposition) \( y \); \( x \) could be ♩, ♭, ♭, ♯, ♬, Eb Major, V13, VII6/5, D, T, SD, presto, forte, etc.; \( x \) could be “eighth note”, “natural”, “whole note”, “composite compass”, “major chord”, “diminished fifth”, “diminished chord”, “tonic”, “fifth degree with added thirteen”, “seventh chord on first inversion”, “metronome velocity”, “intensity of performance”, etc. Examples of exposed signs are part of the musical communication system.

Musical symbols and their concatenations have reference (on the one hand) and intension (on the other hand) have the dual property of referring “objectively” (the work in the absolute sense) and “pragmatically” (the work in a relative sense in a performance concrete)\(^5\).

[...] a construct has a content whereas a signal, such as a written sentence, conveys (carries, transmits) information to someone. In other words, whereas intension is defined on the set of constructs, information is defined on the set of pairs signal-subject, where a subject is a viewer or hearer competent to decode the signal. Information, to be received, requires a receiver, i.e. a system equipped with suitable receiving and decoding devices.\(^6\)

Similarly, we can affirm that the musical grammar intension of a work is defined on the set of constructs and its pragmatic reference is the performance or interpretation of that work. In addition, the information transmitted by the work is referenced by the


signal-subject, where a Subject is a spectator or listener with the ability to decode the signal in its objective reference (in light of concrete musical theories).

The musical sign represents a physical object (tempered note, note value, beat intensity, metronome velocity, etc.) and conceptual (for example the tempered note may be a third major of a chord).

Musical signs are characteristic of the musical notation system and can be linked together to produce more complex signs, and thus their use is a means for communication about certain musical processes, mechanisms or state of musical things.

Any musical symbol written in a bar (or along several bars) designates a concept, and the signs that form a musical phrase (understood as the unit of composition) designate a musical proposition.

In contemporary science theories are expressed by systems of symbols, or symbolisms. What these symbols stand for is more or less clear from (a) the formulas in which they occur and (b) the explicit designation rules that assign constructs to those symbols. An example of a designation rule (or proposal) is: “Let 'S' designate (name, represent, stand for, symbolize) a set”. Such rules are conventional in the sense that the precise symbol chosen to represent a construct is immaterial as long as every time the symbol occurs it be assigned the same construct. The resulting mathematical formalism is not an abstract mathematical theory (e.g. the general theory of groups) but an interpreted one e.g. a theory describing a group of transformations of the Euclidean plane.7

In a similar way we can describe that musical formalism (the syntax of its grammar) is a concrete representation of musical facts and phenomena, this needs to be interpreted to define in a double plane its referents both objectives (rhythmic patterns, metrical patterns in melody and the harmony: all these constitute transformations of the geometric or topological plane) as pragmatic (its performance or performance by the musician-interpreter).

Two types of concepts can be distinguished. One “individual” note is loose for example (or any musical sign) and another “collective” such as the chord (sequence of notes playing simultaneously) for example. Chords are concepts of classes (they refer to types as major, minor, seventh, diminished etc.).

The concept of mathematical function is understood as a correspondence between two sets A and B, where for every member x of A, there is a single element y of B. A propositional musical function relates “individuals” to statements.

7 Ibid, p. 104.
To exemplify; the note “la” (440Hz) can be treated as an $N$ application of a set $O$ of objects, such that, for each individual $i$ contained in $O$, $N(i)$; So for example is the proposition “$i$ is a fifth minor” or “$i$ is a white”. From the above we can write $N: O \rightarrow S$, where the domain $O$ is, for example, the set of notes and $S$ is a set of statements in which the concept $N$ appears.

The isolated note of “la” can be interpreted in many ways, but alone it does not say anything, it can have the figuration of a crotchet or semiquaver etc., can be constituted in tonic (or fundamental), third (major or minor), fifth (Major or minor), sixth, seventh (major or minor), ninth (increased or decreased), etc. You can define a type of chord for example, and from this you can interpret this chord as some degree tonal, and then that tonality as some region of some work, and so on for example.

Scientific concepts are often partitioned into constants and variables. Many of them, whether of the first kind or of the second, are quantitative concepts and most of these are assigned dimensions and units.\(^8\)

The variables that are factually referential can in turn be classed into: (a) object variables, referring to things such as photons or persons; (b) property variables, representing properties of concrete things and relations among them, and (c) spatiotemporal variables, concerning the basic world framework. Variables of all three kinds are likely to occur in one and the same statement.\(^9\)

In music three variables of different nature are simultaneously given, i.e. the musical fact (phrase, chord, etc.) is an object, property and spatiotemporal variable. In the linguistic expression that seeks to refer to the musical phenomena of a concrete work, the three kinds of variables are presented in a single statement.

In a semiotic context, the representational notation of music is a conventional product, this result from the fact that its symbols and rules of construction respond to pragmatic criteria.

**Invariants and their representation in the symbolic musical notation**

Its practical use is due to a simplification and facilitation in the process of being able to reference events of sounds (or the lack of them) that evolve in a space-time continuum on score.

The historical experience produced a conventional construction - on the idea of reference frame - the notion of bar of musical score.

Cooper & Meyer commented that

When we come to the interpretation and analysis of ancient music, the bar of measure is even less reliable as an indicator of the metrical organization. Before the seventeenth century, the bar of measure had no metrical

\(^{8}\) Ibid, p. 40.
\(^{9}\) Ibid, p. 41.
significance that it acquired later. During the Middle Ages and the Renaissance, there was no bar of any kind, hence the placement of the bar in most modern editions was determined by the publisher of the play, not by the composer.¹⁰

This is a pattern that creates a space - in a mathematical sense - potentially divisible into isomorphic or isotopic metrics. The latter - in common practice - assumes quantitative values that make it regular, converting the spatial continuum into a discrete space that generally assumes the values of 2/4, 3/4, 4/4, 5/4, 6/8, 7/8 etc. The denominator indicates a relative unit of duration and the numerator a homogenous isotopic grid.

The key concept in this research is that of “invariant”, and its meaning can be understood in a mathematical sense. In this context, “invariant” is an object or something, that when applied a set of transformations, this object or something does not undergo changes. Thus the entity undergoing transformations must retain its basic relations as an identifiable structure, making it indistinguishable from another original.¹¹

Take as a sample, to follow this research, a real audio played by a computer music sequencer.

This sample shows the sinusoidal waves that make the physical structure of the first bars of guarania "Recuerdos de Ypacarai" by the Paraguayan composer Demetrio Ortiz.

As you can see, the real time almost reaches the exact 6 seconds.

By squaring this interpretation into the rigid of a musical beat of 6/8 time, the following sample is obtained in a metronome time of 90 bpm. As can be seen, the numbers indicate the division of the bars, and the grids correspond to the times of eighth notes, thus have 6 grids for each bar.

\[
\begin{array}{cccccccc}
\text{Figure # 4}
\end{array}
\]

Here is an example of a 'correct' division of “Recuerdos de Ypacarai” in 6/8:

\[
\begin{array}{c}
\text{Figura #5}\textsuperscript{12}
\end{array}
\]

The audio line above corresponds exactly to Figure # 5.

Now, take the audio and insert it into a compas of 2/4 to 60 bpm of metronome time. The following result is obtained. In this integration of the audio to its formal scheme, the numbers correspond to 3 bars, and the grids are distributed in two per bar.

\[
\begin{array}{c}
\text{Figura #5}\textsuperscript{12}
\end{array}
\]

\textsuperscript{12} SANCHEZ HAASE, Diego: La Música en el Paraguay, El lector, Asunción, 2002, pp. 59-64.
The following example in 2/4 is equivalent representation of the previous one in 6/8

![Figure # 6](image6.png)

The previous audio corresponds to the transcription in score of figure # 7.

According to Diego Sánchez Haase, paraphrasing others - like being the teacher Florentín Giménez (1997)- the Guarania should be interpreted in 6/8, otherwise, for example, to write it in 3/4, it would generate some problems that we are going to analyze next.

Before proceeding, it is necessary to clarify a question of referential opacity. The word “interpretation” is extremely ambiguous. In the speech of the cited author, the meaning he acquires is not very clear. The meaning it once suggests refers to the performance of the musician and on another occasion refers to the fact of translating an idea or writing it into a score, therefore the term adds a blurred component to the semantic margins in the speech of the mentioned\(^{13}\).

The first problem that arises - for example - when transcribing a guarania of beginning acephalus inside a measurement of 3/4 is that the silence of eighth note of the beginning in 6/8 (example # 1) is replaced by the one of crotchet in the 3/4 (Figure # 8).

---

\(^{13}\) On interpretation and truth can be consulted BUNGE, Mario: *Semantics II: Interpretation and Truth*, D. Reidel Publishing Company, Dordrecht, 1974b.
In this way, the whole invariant group that forms the melody, loses its syncopation\textsuperscript{14}. This linear displacement that occurs on the space-time axis, coincides the lines of the bass with the first note of each group of eighth notes [on the sense of hearing and the psychological aspects of the musical metric can be consulted London, (2004)]

![Figure # 8](image)

A second problem, taking as a starting point the preservation of the silence that gives rise to the melody, may result in falsely accentuating the second and third beat of the measure.

![Figure # 9](image)

Figure # 9 can be physically illustrated by integrating the audio in example # 5 into a score like the previous time 3/4 at 90 bpm in the following chart. In this integration of the audio to its formal scheme, the numbers correspond to 3 bars, and the grids are distributed in three per bar\textsuperscript{15}.

![Figure # 10](image)

\textsuperscript{14} On metric and rhythm can be consulted KRAMER, Jonathan: The time of music, Schirmer books, New York, 1988, pp. 88 et seq.

\textsuperscript{15} Remember that in the previous example, although it was divided into 3 bars, the grid had a division of 2.
On the basis of these arguments, we can say that, as the author points out, there is a transformation of Figure # 5 in 6/8 in the strict conservation of the whole group invariant (both melody and accompaniment), as the melody is displaced a metric unit of crotchet in ¾.

The interaction between rhythm and meter is complex. On the one hand, the objective organization of a piece of music - temporal relationships, melodic and harmonic structure, dynamics, etc. - creates accents and weak (non-accent) parts and defines their relations. And those accents and non-accents, when they occur with a certain regularity, seem to specify the compass. In this sense, the elements that produce the rhythm also produce the compass...the changes in the melodic, harmonic and temporal relations may result in metric changes.  

Analysis and discussion

Accepting the petitio principii of the cited author, we have that the melodic figuration translated to the measure of 3/4, produces because of the translator’s ear, a copy that betrays the original invariant metric of the whole group, since there is no replica accurate.

I agree absolutely because if we look from a different perspective it is still an equivalent, but this renunciation of the strict conservation of the whole metric of the integrated group produced a transformation. It is a product of the displacement of melodic figuration, but basically the melody is still invariant with respect to its metric and the accompaniment continues conserving its original metric relations.

These cases are very common, in regards to the displacements, in the practice of developing motifs that the composer has experienced in the techniques of compositional manipulation.

The variations of the motifs are given to generate variety to the material with which one works, thus they are given free rein to be inventive or simply as a resource to not cause boredom to the listener.

Now, what happens if the analyst of a work finds that a motif or a phrase is displaced or transformed with respect to another equivalent within the same composition?

Would the musical analyst accuse the author of misrepresenting his own ideas? By this, I do not mean that our hypothetical bad translator is exhibiting deep academic techniques (he may have them or he may not have them).

With respect to the second problem, I want to recognize another example through an illustration of the composition “Paraguayan Dance # 1” by Agustín Barrios (we are

---

talking about an experienced composer) that is written in two different versions of measurement, one with the parameters of $3/4$ and the other with $3/8$.

Although the sets or groups of figures that compose the pulse of the composition (the melodic rhythm and the harmonic rhythm) respond to the ternary and the two measurements are multiple of that pulsation, both the division of measurement and the pulsation are equivalent in figuration structuring similar harmonic and melodic groups.

But this fact does not mean that the symbolic representation of the spatio-temporal events that are referenced in it, close the possibility of other equivalent representations.

Jose Asuncion Flores (Asunción-Paraguay, 1904 – Buenos Aires-Argentina, 1972), creator of Guarania, music genre of folkloric inspiration of Paraguay) himself had doubted, and was not categorical when he pronounced this fact, because in his practice (for pragmatic reasons) he suggested this measure, but not dogmatically, for he did not even observe it rigorously.

Accentuation is a psychological phenomenon that leads to the subjective grouping of musical figures, generating the constant of a pulse\(^1\), and it is from this that can not be assumed - from the human point of view - an absolute spatio-temporal background, the model that represents (our symbolic musical notation) attempts to be an “objective” representation of such events, but this model can not be erected as the only one of that reality.

What is commonly called “interpretation” in music depends on the sensitivity of the performer to the rhythmic structure of music and its awareness of it; the rhythmic grouping - as Cooper and Meyer say - is a mental, not a physical fact, and this is what makes an interpretation an art, which makes possible different phrases and different interpretations of a musical work. The grouping is –at all architectural levels- product of similarity and difference, proximity and separation of sounds perceived by the senses and organized by the mind\(^2\).

Even in physics such a thing is impossible, since the reference frameworks are not absolute (there is no privileged observer) but relative, depending on the observer and its frames of reference, the only thing that makes this events objective (and that there is physics) is that this situation is invariant anywhere in the universe. Our immediate experience will always have a local environment.

These same conclusions are transposed to our analysis.

\(^1\) On structural metrics and the psychological reality of meter can be consulted TEMPERLEY, David: The Cognition of Basic Musical Structures, MIT press, Massachusetts, 2001, p. 23 et seq.

Conclusions

For example, if in a metric of 4/4 we group crotchet triplets we obtain the subjective pulsation of the ternary but in the writing of binary, the reciprocal one is also valid. If we have a grouping of eighth notes in 6/8, two measurements of this is equal to a measure of 4/4. This is what the ear perceives, and from here to the step of turning it into a symbolic representation is the voluntary decision that assumes the construction of the model that pragmatically adjusts to what is intended to represent it.

At the beginning of the twentieth century, the lack of academic training of the popular musicians in Paraguay, left them unable to understand all these issues of the complex building of the musical notation system. For lack of a deep approach with the system of musical notational, they provided solutions to the writing of Paraguayan popular music in a 4/4 measure –such as Nonón Domínguez- but unfortunately they disjuncted the relation between performance and its representation on paper and the correlation of the dictum “traduttore traditore” (the translator is a traitor).\(^{19}\)

Today in the distance of those first attempts of the last century, in the world of microchips, we are better able to try other solutions. Given the hypothetical fact that the solution to the problem of writing Paraguayan popular music would have been the one given in example nº 2 of this essay, and that fact would have become the only official and possible version for writing popular music Paraguayan (Polca and Guarania as examples) would we be arguing that the correct form is 2/4 and the wrong form in 6/8?

Comparing a musical passage written in 6/8 with another in 2/4, within the context of common sense should lead us to conclude that both passages are not equal.

But these maneuvers of comparison are very light and uncritical many times, since they are only maintained at the peripheral level of the formal meaning of the language used for communication.

The proposition “the star of the dawn is the evening star” –a foundational example of Gottlob Frege's semantic analysis and philosophical analysis- is not a mere formality in the sense that it only says at the bottom that “the star of the dawn is the star of the dawn” or “the evening star is the evening star”, forth is identity of the type \(a = b\) is not an empty statement of empirical content.

In this sense Hans D. Sluga writes “The evening star is the evening star” and

\[ \text{The evening star is the morning star’ both deal with the planet Venus and both are true, but one of the sentences is trivial and the other is not. The problem is resolved by distinguishing between the sense and the reference of an expression. That presumably means that we must distinguish between the sense and the reference of the expressions “the evening star” and “the} \]

Invariant equivalents and their representation in the symbolic musical notation: a case in the musical culture of Paraguay

morning star”. But implicit in the problem is the assumption that on the basis of this difference we must also distinguish between the sense and the reference of the sentences in which they occur.20

The above statement is so, because the statement is informative. It is speaking of a question of facts about the world and being thus it is not simply the self-referenced language of the way a = a.

The following expression 'the morning star is the planet Venus' as much as 'the evening star is the planet Venus' have the same truth value, since both refer to the same object, even though the two statements are grammatically different and express different ideas, both have the same reference.

The above analysis also serves to elucidate what was discussed in this research. Although the musical grammar expressed in the example in 6/8 corresponding to the musical work 'Recuerdos de Ypacarai' is not identical to the one expressed in 2/4, both are equivalent because they refer to the same musical object.

Returning the thread of the analysis and to conclude we can say the formal modelling of real audio, that is, from physical audio to musical writing on the staff, shows that the same musical object (in this case the phrase with its accompaniment) can be represented under different models, but the original object is invariant and remains the same despite its variable written presentation.

This is possible only by going deep in to the structure of the musical content, because as we saw, the peripheral structure shows us that they are not the same, but when looking at the spatial structure of both, we find that their forms are equivalent topological, and so, the musical object of this analysis is an invariant that is referenced in both writings despite not being identical grammatically and with very different senses.

From all of the above, and looking again at the expression that there is a correct way to write x (in this case guarania and can also say polka) and is in 6/8, we find that this statement does not is true, since with the developed examples it was shown that by taking the musical object in its topological structure, that is to say, in what it models as a musical object, and keeping this in its essential relations in another grammatical framework, the resulting phenomenon is that the object musical can be written in different ways, but never the less it will always be the same invariant object.

The confusion that is created from seeing, as a conventional system of writing, that has only the effect of giving us a certain ease and flexibility in the manipulation of some elements, neutralizes our memory, to forget that our symbols were created with the purpose to fulfill the role of representing certain parameters and therefore postulate

that this mediatic instrument is the very thing that was wanted to represent is a leap in semantic ascent not allowed, from the logical possibility of existence of a mental construction is passed to an attribution of the actual ontological existence of that fiction (we swallow the story itself).

That the observer is a key factor in physics became clear after Einstein developed his theories. What makes physics objective is that their theories presuppose, apart from some objective reality, that there are potentially observers who can witness this reality. Thus, this phenomenon institutes the principle of objectivity, and this postulates that before the potential diversity of observers, these can make measurements different from reality, but these measures can be related by rules of correspondence that will make them comparable, and from here, it can be said that the rational reconstruction of the material world is intersubjective.

The epistemological understanding that in the knowledge of science is conjectural, based on some theories that find their clear formulation within models that try to fit the best possible to a hidden reality to the simple experience and observation of the common meaning, noses the conclusion, that while it is meritorious and an effort worthy of trying, building models that provide us with glasses to see better, the quest to contemplate the truth as the last essential redoubt will never be reached.

Thus, with Nelson Goodman, we agree that “the score-inscriptions may not all be true copies of one another; yet all will be semantically equivalent -all performances will be of the same work. Work-preservation but not score-preservation is ensured; and insofar as work-preservation is paramount, and score-preservation incidental, redundancy is tolerable”21.